Types of Mathematical Numbering System (Roman - Decimal - Octal - Quinary- Binary)

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Abstract: **When you type some letters or words and enter them into the computer, the computer will translate and convert these texts into numbers, where computers can only understand the language of numbers. Well, the computer can understand letters, numbers and all symbols and deal with them on the basis of numbers, where these letters and symbols and others represent different numerical values depending on the position in which they are located**.

For example, if you want to write a word in English, for example, in capital letters, and of course the result will be different if you use lowercase letters, the computer will convert those letters into binary numbers consisting of 0 and 1, where that word will be converted to its binary counter.

In this research, we will learn about the types of counting systems that the computer understands, in addition to the ways to convert decimal numbers to their corresponding in those systems and vice versa.

Roman numeral systems were used until about the year 1600 AD, and the reason for the spread of this number system for such a long period is due to two main reasons. It requires little basic knowledge.

Keywords: **number system, Roman numeral systems, binary counter**.

1. INTRODUCTION

Some may say that the Roman numeral system depends in its use on the base 10, as they have used symbols for one, ten, hundred, thousand..etc as follows:

one	ten	hundred	thousand
1	X	C	M

As for the fifty-five, they expressed it with the following symbols:

Five	fifty	five hundred
V	L	D

Thus the use of this last set of symbols partially broke the foundation of the Roman numeral system.

The Romans relied on repetition and addition to represent numbers. For example, the numbers 297 and 1974 can be written in the form:

CCLXXXXVII

MDCCCCLXXIIII

where CCLXXXXVII is equal to 2+5+40+50+200 = 297

Later, the place value of the actual symbol was changed. For example, the value of the number XVI in the previous pronunciation is the same as the value of the number IVX, where a new rule was used, which is subtraction. Instead of writing the number 4 on the image IIII, it was placed on the image IV, and this rule shows that the lower number if it is placed in the left This means that it is subtracted from it, and accordingly, the numbers were written in the Roman number system at that stage as follows

9	IX
40	XL
90	XC
400	CD
900	CM

By analysing the way numbers are represented, it is clear that: The number codes for one, ten, only 100 can precede higher units

I	can only precede	V or X
X	can only precede	L or C
C	can only precede	D or M

That is, (I,X,C) it can only precede the two symbols,(D , M) as they denote larger units.

Another use of this rule may lead to an error. For example, does the number I(XC) mean that it is the same number (IX)C where we find that:

I(XC) = one before ninety = 89

While the number (IX)C = nine before the hundred = 91

This new method led the Romans to shorten time when expressing numbers.

for example :

MDCCCCLXXIIII became MCMLXXIV

Since there are no bananas for the powers of a thousand, the Romans used the dash (Bar) above the symbol to indicate that it is multiplied by a thousand, for example: -

$\bar{x}$ means 100

$\bar{c}$ means 10000

Numerical systems

Decimal numerical system = (Arabic - Indian)

If we look at our current numeral system (the Arabic-Indian numeral system), we find that it is characterized by the following characteristics:

1- There are only ten codes:

0,1,2,3,4,5,6,7,8,9 These symbols are called numbers as they are used to write any number regardless of its value.

2- It uses the decimal numerical base in the sense that ten is the basis of aggregation, where the group of this numerical system is tens, tens of tens (hundreds), tens of hundreds (thousands) and the word ten denotes a unit consisting of ten single units. The number 36, for example, is nine units with three units Each of them is made up of ten and six single units.

3- The principle of the place value of the number is used, for example, the number 3 in the number 43 means 3, and in the number 534 it means three tens, and in the number 3671 it means three thousand ... and so on.

4- It follows the system of plural repetition in expressing any number, for example the number 346 = 6 + 4(10) + 3(100)

5- This system does not follow the repetition of symbols, as we do not write the symbol several times to express the number 20.

(We do not write the symbol ten times to express the number 20, that is clear by using the symbol 2 in place of 20)

6- Zero is used in this system as a place occupant. In fact, in addition to its absolute value, which is estimated to be nothing, it is placed in the cell that has no ones, tens, hundreds, or thousands. For example, the number 3040 means:

3(thousands) + zero (hundreds) + 4 (tens) + zero (ones)

One of the disadvantages of the decimal system is that some ordinary fractions cannot be converted accurately to decimal fractions, such as:

$$\frac{1}{3}, \frac{1}{6}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{12}$$

where we resort to rounding to a certain number of digits or to the so-called circular fractions. For example :

$$\frac{1}{3} = 0{,}333333$$

$$\frac{1}{6} = 0{,}1666666$$

Another example of cases in which we resort to approximation is the square roots of some numbers such as

$$\sqrt{2}\ ,\sqrt{3}\ ,\sqrt{5}$$

which makes us decide that not in all cases can regular fractions replace decimals precisely.

Basics other than ten

And any number W can be expressed as follows:

W = $a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3$ … …. As $a_0 , a_{1\,,} a_2 , a_3$

they are the numbers in the number W in order. And its basic symbols are 0,1,2,3,9000 in the decimal system, for example.

X is the base number or aggregation (5 in the pentagonal system, 10 in the decimal system)

If W is a mixed number, then it can be expressed as follows:

$a_1x^{-1} + a_2x^{-2} + a_3x^{-3}$ … …. As $a_0 , a_{1\,,} a_2 , a_3$

are the base symbols, X is the base and for example:

$$3456 = 6(10^0) + 5(10^1) + 4(10^2) + 3(10^3)$$

2. OCTAL NUMBER SYSTEM

The characteristics of the octal system

1- Eight base assembly

2- The basic symbols used in it are from 0 to 7

3- When writing any number in the octal system, it is written in relation to the powers of eight, which are:

$$8^1 , 8^1 , 8^2 , 8^3 , 8^4 ,,,,,,,$$

4- It has the place value property of a number in a number.

To clarify how to write the symbols of this system, we provide the following some numbers in the decimal system and their equivalent numbers in the octal system:

decimal system	0	1	2	3	4	5	6	7	8	9	10
octal system	0	1	2	3	4	5	6	7	10	11	12

Thus, the addition table in the octal system is as follows:

+	1	2	3	4	5	6	7
1	2						
2	3	4					
3	4	5	6				
4	5	6	7	10			
5	6	7	10	11	12		
6	7	10	11	12	13	14	
7	10	11	12	13	14	15	16

Examples of adding numbers in the octal system

Summation $(42)_8 + (67)_8 = (131)_8$

Example(2) : $(53)_8 + (75)_8 = (130)_8$

Example(3) : $(444)_8 + (777)_8 = (1433)_8$

Subtraction in the octal system

- 7 – 3 = 4
- 6 – 5 = 1
- 13 – 5 = 6

Note: If the numbers included in the subtraction process are less than eight, the result will be as followed in the decimal system.

Example (1)

to find 543 - 267 in the octal system

$$(543)_8 - (267)_8 = (254)_8$$

Example (2)

$$(543)_8 - (177)_8 = (344)_8$$

Example (3)

$$(711)_8 - (466)_8 = (223)_8$$

Multiplication in the octal system

Here is the multiplication table in the octal system:

X	1	2	3	4	5	6	7
1	1						
2	2	4					
3	3	6	11				
4	4	10	14	20			
5	5	12	17	24	31		
6	6	14	22	30	36	44	
7	7	16	25	34	43	52	61

Example (1) :

$$(214)_8 \ * (72)_8 = (17670)_8$$

Example (2) :

$$(161)_8 \ * (57)_8 = (12652)_8$$

Division in the octal system

Example (1)

(15446) / (62)

```
      213
    -----------------
62 |15446
   |144
     --------
     104
     62
     ---------
     266
     266
     ----------
     0000
```

$$\mathbf{(15446)_8 \ / \ (62)_8 = (213)_8}$$

Converting from decimal to octal

Example (1) : 219 =

Solution (1)

$$3(8)^1 + 3(8)^1 + 3(8)^2$$

$$\mathbf{(219)_{10} = (333)_8}$$

Solution (2) 219 =

remainder of the division	number	system
3	219	8
3	27	8
3	3	8
	0	

Example (2)

To convert 763 from decimal to octal

remainder of the division	number	system
3	763	3
7	95	7
3	11	3
1	1	1
	0	

$$\mathbf{(763)_{10} = (1373)_8}$$

Example (3) to check whether

$$\mathbf{(1111)_{10} = (2127)_8}$$

remainder of the division	number	system
7	1111	8
2	138	8
1	17	8
2	2	8
	0	

Example (4) to convert a number $\mathbf{(1111)_8}$ to its equivalent in the decimal system

$$\mathbf{(1111)_8}=1+1(8\ \ )+1(8^2)+1(8^3)+1(8^4)$$

$$1+8+64+512 = 585$$

$$\mathbf{(1111)_8 = (585)_{10}}$$

Converting from the octal system to the decimal system

Example (1) to convert the number 125 from the octal system to the decimal system

$$\mathbf{(125)_8}=5+2(8^1)+1(8^2)$$

$$= 5+16+64 = 85$$

$$\mathbf{(125)_8 = (85)_{10}}$$

Example (2) to convert the number 234 from the octal system to the decimal system

$$\mathbf{(234)_8}=4+3(8^1)+2(8^2)$$

$$= 4+24+128$$

$$= 156$$

$$\mathbf{(234)_8 = (156)_{10}}$$

3. QUINARY SYSTEM

The characteristics of the Quinary system

1. The five aggregation basis
2. The basic symbols used in it are from 0 to 5
3. When writing any number in the Quinary_system, it is written in relation to the powers of five, which are:

$$5^0, 5^1, 5^2, 5^3, 5^4,,,,,,,$$

4. It has the place value property of a number in a number.

To clarify how to write the symbols of this system, we provide the following some numbers in the decimal system and their equivalent numbers in the Quinary :

decimal system	0	1	2	3	4	5	6	7	8	9	10
Quinary_system	0	1	2	3	4	10	11	12	13	14	20

Thus, the addition table in the Quinary_system is as follows:

+	1	2	3	4	5
1	2				
2	3	4			
3	4	10	11		
4	10	11	12	13	
5	11	12	13	14	20

Examples of adding numbers in the Quinary system

Example(1) $(243)_5 + (341)_5 = (1134)_5$

Example(2) $(332)_5 + (421)_5 = (1303)_5$

Subtraction in the Quinary system

- 4 – 1 = 3
- 3 – 3 = 0
- 10 – 4 = 1
- 11 – 4 = 2
- 14 – 4 = 10
- 13 – 4 = 4

Note: If the numbers included in the subtraction process are less than five, the result will be as followed in the decimal system

$$(341)_5 - (324)_5 = (12)_5$$

$$(4222)_5 - (1333)_5 = (2334)_5$$

Multiplication in the Quinary system

Here is the multiplication table in the Quinary_system:

*	1	2	3	4
1	1			
2	2	4		
3	3	11	14	
4	4	13	22	31

Example (1) :

$$(343)_5 * (32)_5 = (23131)_5$$

Example (2) :

$$(401)_5 * (32)_5 = (23332)_5$$

Division in the Quinary system

Example (1)

$$(1034)_5 \; / \; (22)_5$$

```
         22
      -----------------
 22 | 1034
    | 44
       --------
       44
       44
       ------
       0000
```

$$(1034)_5 \; / \; (22)_5 = (22)_5$$

Converting from decimal to Quinary

Example (1) converting the number 444 from the decimal system to the Quinary :

remainder of the division	number	system
4	444	5
3	88	5
2	17	5
3	3	5
	0	

$$(444)_{10} = (3234)_5$$

Example (2) converting the number 234 from the decimal system to the Quinary :

remainder of the division	number	system
4	234	5
1	46	5
4	9	5
1	1	5
	0	

$$(234)_{10} = (1414)_5$$

Example (3) to convert a number $(\mathbf{143})_5$ to its equivalent in the decimal system

$$(\mathbf{143})_5 = \mathbf{3} + \mathbf{4} * (\mathbf{5}) + \mathbf{1} * (5^2)$$

$$= 3 + 20 + 25$$

$$= 48$$

$$(134)_5 = (48)_{10}$$

4. BINARY COUNTING SYSTEM

The characteristics of the Binary system

1. The basis of the compilation of the two .
2. The basic symbols used in it are from 0 to 1
3. When writing any number in the Binary system, it is written in relation to the powers of two, which are:

$$2^0, 2^1, 2^2, 2^3, 2^4, , , , , , ,$$

4. It has the place value property of a number in a number.

To clarify how to write the symbols of this system, we provide the following some numbers in the decimal system and their equivalent numbers in the Binary :

decimal system	0	1	2	3	4	5	6	7	8	9	10
Binary system	0	1	10	11	100	101	110	111	1000	1001	1010

Thus, the addition table in the pentagonal system is as follows:

+	0	1
0	0	1
1	1	11

Examples of adding numbers in the Binary system

Example(1) $(11)_2 + (11)_2 = (110)_2$

Example(2) $(1101)_2 + (101)_2 = (10010)_2$

Subtraction in the Binary system

Example(1) $(101)_2 - (11)_2 = (10)_2$

Example(2) $(1000)_2 - (101)_2 = (11)_2$

Multiplication in the pentagonal system

Example (1) :

$$(111)_2 * (11)_2 = (10101)_2$$

Example (2) :

$$(101)_2 * (101)_2 = (11001)_2$$

Division in the Binary system

Example (1)

$$(11001)_2 \ / \ (101)_2$$

```
        101
     ----------------
 101 | 11001
     |  101
       --------
       101
       101
       ------
       0000
```

$$(11001)_2 \ / \ (101)_2 = (101)_2$$

Converting from decimal to Binary

Example (1) converting the number 100 from the decimal system to the Binary :

remainder of the division	number	system
0	100	2
0	50	2
1	25	2
0	12	2
0	6	2
1	3	2
1	1	2

$$(100)_{10} = (1100100)_2$$

Example (2) converting the number 135 from the decimal system to the Binary :

remainder of the division	number	system
1	135	2
1	67	2
1	33	2
0	16	2
0	8	2
0	4	2
0	2	2
1	1	2

$$(135)_{10} = (10000111)_2$$

Converting from Binary to Decimal

Example (1)

to convert a number $(\mathbf{1011})_2$ to its equivalent in the decimal system

$$(\mathbf{1011})_2 = \mathbf{1} + \mathbf{1} * (\mathbf{2}) + \mathbf{0} * \left(2^2\right) + \mathbf{1} * \left(2^3\right)$$

$$= 1 + 2 + 0 + 8$$

$$= 11$$

$$(1011)_2 = (11)_{10}$$

Example (2)

to convert a number $(\mathbf{11111})_2$ to its equivalent in the decimal system

$$(\mathbf{11111})_2 = \mathbf{1} + \mathbf{1} * (\mathbf{2}) + \mathbf{1} * \left(2^2\right) + \mathbf{1} * \left(2^3\right) + \mathbf{1} * \left(2^4\right)$$

$$= 1 + 2 + 4 + 8 + 16$$

$$= 31$$

$$(11111)_2 = (31)_{10}$$